ON THE THEORY OF RAYLEIGH'S INSTABILITY

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It is well known that the equilibrium of a fluid contained in a plane vertical layer or channel and heated from below becomes unstable, when the temperature gradient reaches its critical value. We find [1] that perturbations, velocity vectors of which are parallel to the generators of the channel, correspond to a minimum value of the Rayleigh number and are most susceptible to the instability effect. Cellular perturbations periodic with respect to the axis of the channel, correspond to higher values of the Tayleigh number [2 and 3].

The situation is different in the case of plane horizontal layers or channels. When we have a horizontal layer heated from below, we find that the onset of instability is caused by perturbations possessing a definite wavelength [4], and periodic in the plane of the layer. In the case of a horizontal circular cylinder, experiments [5] and calculations [6] have shown that the appearance of instability is also connected with cellular perturbations.

Below we investigate how the form of instability changes with inclination of the channel relative to the vertical and we study the convective stability of a plane layer arbitrarily oriented with respect to the force of gravity. It appears that at some critical value α_0 of the angle of inclination of the plane of the layer to the vertical, the instability changes its character (transition from the plane-parallel to the cellular perturbations). When $\alpha > \alpha_0$, then critical perturbations possess a finite wavelength.

A method of small parameter based on expansion of perturbations into power series in terms of a dimensionless wave number k, was used in studying long-wave perturbations. This yielded spectra of perturbations and of the critical Rayleigh numbers at small k, and made it possible to determine the critical angle α_0. For a layer bounded by perfectly heat-conducting planes, $\alpha_0 = 21°$.

Lower instability levels at arbitrary wave numbers were investigated with help of the Galerkin method. The boundary value problem for perturbation amplitudes is reduced to a system of homogeneous linear algebraic equations for the coefficients of the expansion, and its matrix is numerically diagonalized on the digital computer. Eigenvalues, i.e. the critical values of the Rayleigh number were found for various values of α and k.

1. Perturbation equations. A plane infinite layer of thickness $2h$ inclined at an angle α to the vertical (Fig. 1), is heated from below in such a manner, that a state of equilibrium is possible. During the equilibrium the fluid is at rest. The temperature T_0 and the pressure p_0 (with respect to hydrostatic pressure at constant density ρ_0) are given by

$$\nabla T_0 = -A\gamma, \qquad \nabla p_0 = \rho_0 g\beta T_0\gamma \tag{1.1}$$

where γ is the unit vertical vector directed upwards, A is the constant equilibrium temperature gradient ($A > 0$ corresponds to heating from below), while g and β are the acceleration due to gravity and the coefficient of thermal expansion, respectively.

Usual convection equations yield equations of "neutral" perturbations

$$-\frac{\nabla p}{\rho_0}+\nu\Delta \mathbf{v}+g\beta T\boldsymbol{\gamma}=0, \qquad \mathbf{v}\nabla T_0=\chi\Delta T, \qquad \operatorname{div}\mathbf{v}=0 \tag{1.2}$$

Here ν and χ are the coefficients of kinematic viscosity and heat conductivity, respectively. We can rewrite these equations in a dimensionless form, using h, χ/h, Ah and $\rho_0\nu\chi/h^2$ as the units of distance, velocity, temperature and pressure, respectively. Then (1.2) can be replaced by dimensionless equations for velocity, temperature and pressure perturbations denoted, as before, by $\mathbf{v}$, T and p

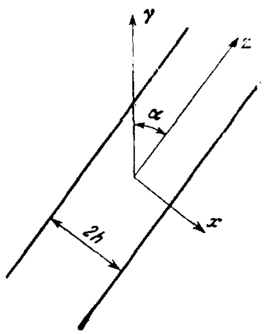

Fig. 1

$$\Delta\mathbf{v}+RT\boldsymbol{\gamma}=\nabla p, \qquad R=g\beta Ah^4/\nu\chi \tag{1.3}$$

$$\Delta T=-(\mathbf{v}\boldsymbol{\gamma}), \qquad \operatorname{div}\mathbf{v}=0 \tag{1.4}$$

where the Rayleigh number R is a dimensionless parameter.

In the following we shall consider only such plane perturbations, for which velocity components v_x and $v_z \neq 0$, $v_y=0$ and all magnitudes are independent of y. In this case we can introduce a stream function

$$v_x=\partial\psi/\partial z, \quad v_z=-\partial\psi/\partial x \tag{1.5}$$

Taking curl of (1.3) to eliminate the pressure component and introducing the stream function, we obtain

$$\Delta\Delta\psi-R\left(\sin\alpha\frac{\partial T}{\partial z}+\cos\alpha\frac{\partial T}{\partial x}\right)=0, \quad \Delta T-\left(\sin\alpha\frac{\partial\psi}{\partial z}+\cos\alpha\frac{\partial\psi}{\partial x}\right)=0 \tag{1.6}$$

"Normal" perturbations periodic in the z-direction, can be written in the form

$$\psi(x,z)=\varphi(x)e^{ikz}, \qquad T(x,z)=\theta(x)e^{ikz} \tag{1.7}$$

where k is the real wave number. Inserting (1.7) into (1.6) we obtain Eqs. for amplitudes of perturbations $\varphi(x)$ and $\theta(x)$

$$\varphi^{IV}-2k^2\varphi''+k^4\varphi=R(ik\sin\alpha\,\theta+\cos\alpha\,\theta') \tag{1.8}$$

$$\theta''-k^2\theta=ik\sin\alpha\,\varphi+\cos\alpha\,\varphi' \tag{1.9}$$

Both velocity components together with the temperature perturbation disappear on the boundaries of the layer (last condition corresponds to the case of perfectly conducting walls). Boundary conditions of (1.8) and (1.9) then become

$$\varphi=\varphi'=\theta=0 \qquad \text{when } x=\pm 1 \tag{1.10}$$

The problem (1.8) to (1.10) is characteristic, yielding the critical values of the Rayleigh number R (for given α and k) and the corresponding critical motions.

2. Long wave perturbations. Critical angle. If the wave of a perturbation is large when compared with the thickness of the layer, then the dimensionless wave number k is small and the method of small parameter can be used to obtain a solution to our problem.

We shall seek the amplitudes of perturbations φ and θ and the critical Rayleigh number R expressed as power series in terms of a small parameter (ik)

$$\varphi=\varphi^{(0)}+(ik)\varphi^{(1)}+(ik)^2\varphi^{(2)}+\ldots \tag{2.1}$$

$$\theta = \theta^{(0)} + (ik)\,\theta^{(1)} + (ik)^2\theta^{(2)} + \ldots \tag{2.2}$$

$$R = R^{(0)} + k^2R^{(2)} + k^4R^{(4)} + \ldots \tag{2.3}$$

(since R is real, (2.3) obviously contains only even powers of k). Equations of consecutive approximations are

$$\varphi^{(0)\mathrm{IV}} - R^{(0)}\cos\alpha\,\theta^{(0)'} = 0, \qquad \theta^{(0)''} - \cos\alpha\,\varphi^{(0)'} = 0 \tag{2.4}$$

$$\varphi^{(1)\mathrm{IV}} - R^{(0)}\cos\alpha\,\theta^{(1)'} = R^{(0)}\sin\alpha\,\theta^{(0)}. \quad \theta^{(1)''} - \cos\alpha\,\varphi^{(1)'} = \sin\alpha\,\varphi^{(0)} \tag{2.5}$$

$$\varphi^{(2)\mathrm{IV}} - R^{(0)}\cos\alpha\,\theta^{(2)'} = -2\varphi^{(0)''} + R^{(0)}\sin\alpha\,\theta^{(1)} - R^{(2)}\cos\alpha\,\theta^{(0)} \tag{2.6}$$

$$\theta^{(2)''} - \cos\alpha\,\varphi^{(2)'} = -\theta^{(0)} + \sin\alpha\,\varphi^{(1)} \quad \text{and so on.}$$

The boundary conditions coincide, for all approximations, with (1.10).

Zero approximation (2.4) gives critical values $R^{(0)}$ and corresponding amplitudes for plane-parallel perturbations ($k = 0$). Homogeneous system (2.4) defines, together with boundary conditions (1.10), two classes of solutions. Solutions belonging to the first class ("odd" solutions) will have both, velocity $v_z^{(0)}$ and the temperature $\theta^{(0)}$, appearing as odd functions of x, and will have the form

$$\varphi^{(0)} = \frac{\cos\gamma x}{\cos\gamma} - 1, \quad \theta^{(0)} = \frac{\cos\alpha}{\gamma}\,\frac{\sin\gamma x}{\cos\gamma} \tag{2.7}$$

$$\gamma \equiv (R^{(0)}\cos^2\alpha)^{1/4} \tag{2.8}$$

A relation characteristic to odd solutions, leads to the following spectrum of odd instability levels

$$\gamma = n\pi, \qquad R^{(0)} = \frac{\gamma^4}{\cos^2\alpha} = \frac{n^4\pi^4}{\cos^2\alpha} \qquad (n = 1, 2, 3\ldots) \tag{2.9}$$

The amplitudes of even solutions are

$$\varphi^{(0)} = \frac{\sin\gamma x}{\sin\gamma} - \frac{\mathrm{sh}\,\gamma x}{\mathrm{sh}\,\gamma}, \qquad \theta^{(0)} = \frac{\cos\alpha}{\gamma}\left[\frac{\cos\gamma - \cos\gamma x}{\sin\gamma} + \frac{\mathrm{ch}\,\gamma - \mathrm{ch}\,\gamma x}{\mathrm{sh}\,\gamma}\right] \tag{2.10}$$

Here γ is related to $R^{(0)}$ by (2.8), but in the case of even solutions the values of γ are given by the roots of the following Eq.

$$\mathrm{tg}\,\gamma = \mathrm{th}\,\gamma \qquad (\gamma = 3.927,\ 7.069, \ldots) \tag{2.11}$$

Similarly, we find the critical levels $R^{(0)}$ for even perturbations

$$R^{(0)} = \frac{\gamma^4}{\cos^2\alpha} = \frac{(3.927)^4}{\cos^2\alpha}, \qquad \frac{(7.069)^4}{\cos^2\alpha}, \ldots \tag{2.12}$$

Odd and even levels in the spectrum of critical values of γ (and hence of R^0) alternate in the following manner:

$$\gamma_1 = \pi, \quad \gamma_2 = 3.927, \quad \gamma_3 = 2\pi, \quad \gamma_4 = 7.069, \ \ldots \tag{2.13}$$

Consecutive approximation equations must be used to find the corrections to the levels and amplitudes at small values of k. A nonhomogeneous system must be solved in each approximation, and the condition of solvability of this system defines the corresponding correction to the nonperturbed critical value.

First order amplitude corrections are given by (2.5). On inspection of the right-hand sides of (2.5) we can easily see that the parity of first order corrections is opposite to that of the zero order amplitudes, and this is true for all odd order corrections. Even order amplitude corrections will, on the other hand, have the same parity as that of a zero approximation. From this it follows that the solution has, as a whole, no definite parity when $k \neq 0$, and this can also be seen directly from the system (1.8), (1.9).

Let us write the higher order corrections to the amplitudes of odd solutions

$$\varphi^{(1)} = \operatorname{tg}\alpha\left[\frac{x\cos\gamma x}{2\cos\gamma} + \frac{3(\gamma\operatorname{cth}\gamma - 1)}{2\gamma\cos\gamma}\sin\gamma x - \frac{3}{2}\frac{\operatorname{sh}\gamma x}{\operatorname{sh}\gamma} + x\right]$$
$$\theta^{(1)} = \sin\alpha\left[\frac{x\sin\gamma x}{2\gamma\cos\gamma} + \frac{(2-3\gamma\operatorname{cth}\gamma)}{2\gamma^2\cos\gamma}\cos\gamma x - \frac{3\operatorname{ch}\gamma x}{2\gamma\operatorname{sh}\gamma} + \frac{3\gamma\operatorname{cth}\gamma - 1}{\gamma^2}\right] \tag{2.14}$$

The values of γ are given by (2.9). The corresponding formulas for the first order amplitude corrections in the case of even solutions, are

$$\varphi^{(1)} = \frac{\operatorname{tg}\alpha}{2}\left[\frac{x\sin\gamma x}{\sin\gamma} - \frac{x\operatorname{sh}\gamma x}{\operatorname{sh}\gamma} + \frac{1+3\gamma\operatorname{ctg}\gamma}{\gamma\sin\gamma}\cos\gamma x + \right.$$
$$\left. + \frac{1+3\gamma\operatorname{ctg}\gamma}{\gamma\operatorname{sh}\gamma}\operatorname{ch}\gamma x - \frac{2}{\gamma}\operatorname{ctg}\gamma(1+3\gamma\operatorname{ctg}\gamma)\right] \tag{2.15}$$
$$\theta^{(1)} = \sin\alpha\left[\frac{3\operatorname{ctg}\gamma}{2\gamma\sin\gamma}\sin\gamma x + \frac{3\operatorname{ctg}\gamma}{2\gamma\operatorname{sh}\gamma}\operatorname{sh}\gamma x - \frac{x\cos\gamma x}{2\gamma\sin\gamma} - \frac{x\operatorname{ch}\gamma x}{2\gamma\operatorname{sh}\gamma} - \frac{2\operatorname{ctg}\gamma}{\gamma}x\right]$$

where values of γ are given by (2.11).

Zero and first order amplitudes can be used to find the second order correction $R^{(2)}$. This is obtained from the condition of solvability of a nonhomogeneous system (2.6) for the second order amplitudes

$$R^{(2)}\cos\alpha\int_{-1}^{1}\varphi^{(0)}\theta^{(0)\prime}\,dx = \int_{-1}^{1}[2\varphi^{(0)\prime}\varphi^{(0)\prime} + R^{(0)}\theta^{(0)}\theta^{(0)} +$$
$$+ R^{(0)}\sin\alpha(\varphi^{(1)}\theta^{(0)} - \varphi^{(0)}\theta^{(1)})]\,dx \tag{2.16}$$

Insertion of previously obtained values of zero and first order amplitudes into (2.16) yields, for odd levels,

$$R^{(2)} = \frac{3\gamma^2}{\cos^2\alpha}\left[1 - \frac{1}{2}(6\gamma\operatorname{cth}\gamma - 5)\operatorname{tg}^2\alpha\right] \tag{2.17}$$

The behavior of neutral curves $R(k)$ is, for small k, defined by the sign of $R^{(2)}$. If $R^{(2)} > 0$, then the stability curve exhibits a minimum at $k = 0$, while when $R^{(2)} < 0$ we have a maximum. We see from (2.17) that, for small α, $R^{(2)}$ is positive; it decreases monotonously with increasing α and at some α_0 it changes its sign. Critical angle α_0 is defined by the condition $R^{(2)} = 0$, from which we obtain

$$\alpha_0 = \operatorname{arctg}\left(\frac{2}{6\gamma\operatorname{cth}\gamma - 5}\right)^{1/2} \tag{2.18}$$

The lowest odd level defines the threshold of convection. It is the fundamental level of the spectrum and the corresponding value of γ is $\gamma = \pi$ (see (2.9). Then, (2.18) yields $\alpha_0 = 20°46'$.

Thus, within the interval $0 < \alpha < \alpha_0$, the minimum Rayleigh number corresponds to plane parallel perturbations when $k = 0$ and is equal, in accordance with (2.9), to

$$R_{1*} = \frac{\pi^4}{\cos^2\alpha} \tag{2.19}$$

When $\alpha > \alpha_0$, then the point $k = 0$ corresponds to a maximum on the curve $R_1(k)$, the minimum is displaced into the region $k \neq 0$ and the corresponding critical value of R is found numerically as shown in Section 3.

The behavior of the instability curves $R(k)$ in the region of small k for higher order odd levels is completely analogous to the behavior of the fundamental level which we have discussed above. All critical angles are obtained from (2.18) by the substitution $\gamma = n\pi$. Clearly, the value of α_0 decreases with increasing n. Putting $\gamma = 2\pi$ in (2.18) we find, for the second odd level (third level R_3 in the whole spectrum), $\alpha_0 = 13°53'$.

The behavior of even instability levels is established by inserting the values of zero and first order approximations given for the even amplitudes by (2.10) and (2.15), into (2.16). The resulting formula for $R^{(2)}$ analogous to (2.17) is very unwieldy and is, therefore, not given here. However we can deduce from it, that the second order correction to $R^{(2)}$ is positive for all α. Thus, for all α, the neutral curves $R(k)$ of even levels have a minimum at $k=0$, and the corresponding minimum values of R_* are given by (2.12). It seems, that the method of small parameter leaves the question of uniqueness of the minimum at $k=0$, open. To settle it, we must consider the stability under perturbations with a finite k (see Section 3). Nevertheless, we can already infer that a minimum should exist for finite k, in any case, for angles near to 90°. Indeed, as $\alpha \to 90°$, we obtain the Rayleigh problem of stability of a horizontal layer in which, as we know, minimum critical values of the Rayleigh number correspond at all levels of instability, to finite wavelengths.

3. **Numerical results.** The complete system (1.8) and (1.9) must be used to obtain the spectrum of instability at finite values of the wave number k. A general solution of a linear system with constant coefficients (1.8) and (1.9) can be written out, but the resulting characteristic equation from which critical Rayleigh numbers must be obtained, is very complex. Therefore an approximate method due to Galerkin is found to be more rewarding. To use it, we shall represent the amplitudes of the stream function and temperature in the form

$$\varphi = a_0\varphi_0 + a_1\varphi_1 + a_2\varphi_2 + \ldots, \qquad \theta = b_0\theta_0 + b_1\theta_1 + b_2\theta_2 + \ldots \tag{3.1}$$

where the eigenfunctions of the following boundary value problems

$$\varphi_i^{\mathrm{IV}} - 2k^2\varphi_i'' + k^4\varphi_i = -\mu_i(\varphi_i'' - k^2\varphi_i), \qquad \varphi_i = \varphi_i' = 0 \text{ when } x = \pm 1 \tag{3.2}$$

$$\theta_l'' - k^2\theta_l = -\nu_l\theta_l \qquad \theta_l = 0 \quad \text{when } x = \pm 1 \tag{3.3}$$

play the part of the base functions φ_i and θ_i.

They are given in their explicit form together with relations for the eigen numbers in [7], where the above base was used in investigating the spectrum of normal perturbations of a steady convective motion.

Inserting (3.1) into (1.8) and (1.9), multiplying first of the resulting expressions by φ_i and the second one by θ_l and integrating with respect to x from -1 to $+1$, we obtain an infinite linear homogeneous system of equations of the Galerkin method for the coefficients a_i and b_i.

Equating the determinant of this system to zero, we obtain a characteristic equation defining the critical values of R as functions of α and k. We can write this equation as

$$\begin{vmatrix} (a) & (b) \\ (c) & (d) \end{vmatrix} = 0 \tag{3.4}$$

where (a), (b), (c) and (d) are matrices whose general terms are given by

$$a_{mn} = -R^{-1} I_m \delta_{mn}, \qquad b_{mn} = ik \sin\alpha\, C_{mn} + \cos\alpha D_{nm}$$

$$c_{mn} = ik \sin\alpha C_{nm} - \cos\alpha D_{mn}, \qquad d_{mn} = {}^1/_2 \nu_m \delta_{mn} \tag{3.5}$$

Here δ_{mn} is a Kronecker delta, while matrix elements C_{mn}, D_{mn} and I_m have values depending on signs of the indices

$$C_{mn} = (-1)^{1/2\, m} f_{mn} \quad \text{(when } m \text{ and } n \text{ are even)}$$
$$C_{mn} = (-1)^{1/2(m-1)} f_{mn} \quad \text{(when } m \text{ and } n \text{ are odd)}$$
$$C_{mn} = 0 \quad \text{(when } m \text{ and } n \text{ are of opposite signs)}$$

$$D_{mn} = (-1)^{1/2(m+2)} f_{mn}\, k \operatorname{cth} k \quad \text{(when } m \text{ is even and } n \text{ is odd)}$$
$$D_{mn} = (-1)^{1/2(m+1)} f_{mn}\, k \operatorname{th} k \quad \text{(when } m \text{ is odd and } n \text{ is even)}$$
$$D_{mn} = 0 \quad \text{(when } m \text{ and } n \text{ are of the same sign)}$$

$$I_m = \frac{\mu_m^2}{2(\mu_m - k^2)} (\mu_m - k^2 - k \operatorname{th} k + k^2 \operatorname{th}^2 k) \quad \text{(when } m \text{ is even)}$$
$$I_m = \frac{\mu_m^2}{2(\mu_m - k^2)} (\mu_m - k^2 - k \operatorname{cth} k + k^2 \operatorname{cth}^2 k) \quad \text{(when } m \text{ is odd)}$$
$$f_{mn} = \frac{\pi (m+1) \mu_n}{2\nu_m(\mu_n - \nu_m)} \tag{3.6}$$

First eight terms were retained in each expansion of (3, 1). Under these conditions the characteristic Eq. (3, 4), the left-hand side of which was a 16th order determinant, yielded eight levels of the spectrum of critical values of R. However only the lowest levels were found to be sufficiently accurate.

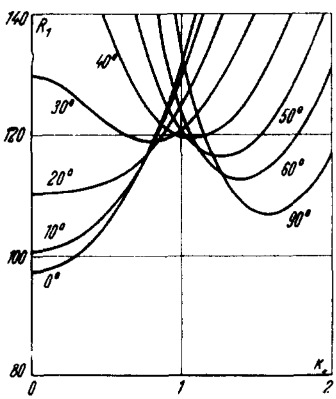

Fig. 2

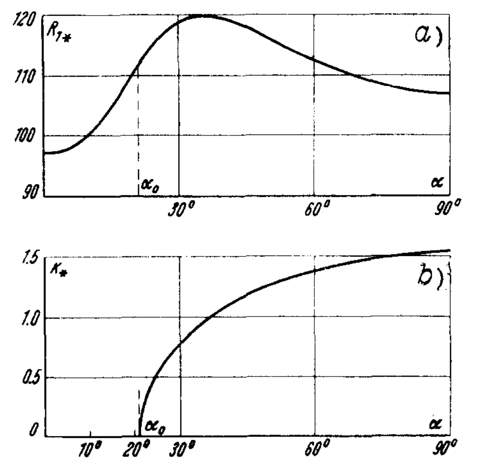

Fig. 3

Orthogonal step method [8] was used to diagonalize the matrix. Actual computation was performed on the "Aragats" digital computer (here thanks are due to S. Keller and A. Koblov for help in performing the computations).

Let us now discuss the obtained results.

Fig. 2 gives the neutral curves $R_1(k)$ of the fundamental instability level for various angles of inclination of the layer towards the vertical. When $0 \leq \alpha < 21°$, the critical values of R_1 increase monotonously with k, and a minimum corresponds to $k = 0$. When $\alpha > 21°$, then in accordance with Section 2 the minimum is displaced into the region $k \neq 0$. Fig. 3a gives the minimum value R_{1*} defining the limit of stability relative to the angle of inclination α. We see from it that the stability reaches its maximum in the region of $\alpha = 35°$. When $\alpha = 90°$ (plane horizontal layer heated from below (*), $R_{1*} = 106.8$ and the corresponding wave number $k_{1*} = 1.56$.

An interesting fact emerges from our discussion, namely, that the critical number R_{1*} is weakly dependent on the angle of inclination. However, during the change of inclination the instability alters its character; when

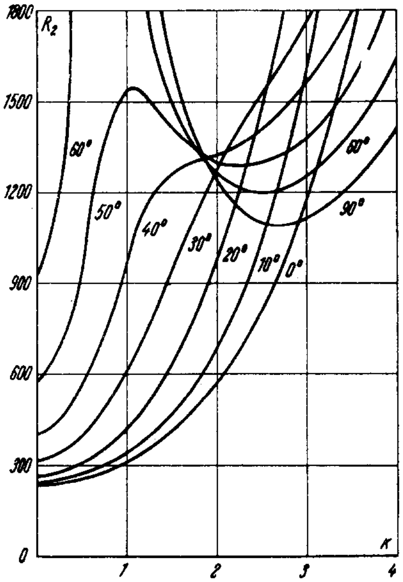

Fig. 4

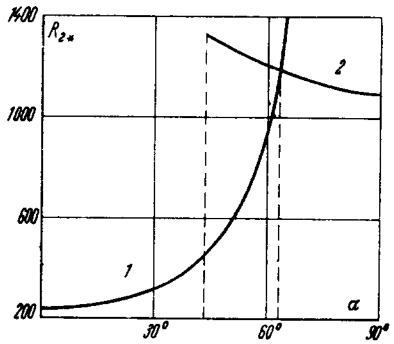

Fig. 5

$\alpha < 21°$, the instability is caused by plane parallel motions ($k = 0$), while when $\alpha > 21°$, the instability appears in the form of Benard cells whose wave number k_* increases monotonously with α (Fig. 3b).

We shall now investigate the behavior of k_* near the critical angle α_0, using (2.17). With (2.18) taken into account, we can write the second order correction $R^{(2)}$ as

$$R^{(2)} = \frac{3\gamma^2(6\gamma \operatorname{cth} \gamma - 5)}{2\cos^2\alpha} (\operatorname{tg}^2 \alpha_0 - \operatorname{tg}^2 \alpha) \tag{3.7}$$

When α is nearly equal to α_0, we have

$$R^{(2)} = a(\alpha_0 - \alpha) \tag{3.8}$$

Expansion (2.3) of the critical Rayleigh number up to and including k^4, is

$$R = R^{(0)} - a(\alpha - \alpha_0) k^2 + R^{(4)} k^4 \qquad (a > 0,\ R^{(4)} > 0) \tag{3.9}$$

Minimizing R in k we now find that

$$k_* = \left(\frac{a}{2R^{(4)}} (\alpha - \alpha_0)\right)^{1/2} \tag{3.10}$$

i.e. k_* increases as a square root when $\alpha \gtrless \alpha_0$.

Next we shall consider the second instability level R_2. The neutral curves $R_2(k)$ are given, for various α, in Fig. 4. In accordance with the results obtained by the method of small parameter (Section 2), a minimum now exists on stability curves at the point

*) It should be noted that we have used half of the width of the layer in determining the Rayleigh number. If total width and temperature difference are used, then the value quoted above should be multiplied by 16, and this yields $R_{1*}^{+} = 1709$ which is in good agreement with [4].

$k = 0$ for all values of α. The corresponding minimum value R_{2*} varies with α according to (2.12) (first root) and is shown in Fig. 5 by the curve 1. With increasing α, a point of inflection appears on stability curves, and is followed by a second minimum at $\alpha = 43°$

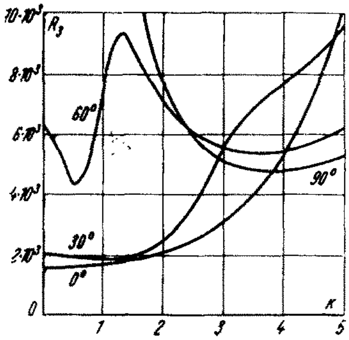

Fig. 6

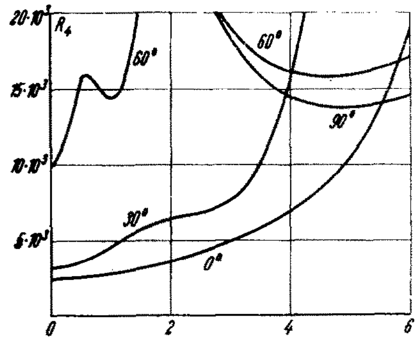

Fig. 7

and $k = 2.0$. When α increases further, this minimum shifts towards the higher wave numbe.... Fig. 5 curve 2 shows the dependence of the corresponding minimum critical number R_{2*} on α. The critical wave number k_* increases monotonously along the curve 2, from 2.0 at $\alpha = 43°$ to 2.7 at $\alpha = 90°$. The change of the instability character (transition from plane parallel to the cellular motion) takes place at $\alpha = 63°$ at the point of intersection of the curves 1 and 2, where k_* changes discontinuously from zero to some finite value.

In the limiting case when $\alpha = 90°$, $R_{2*} = 1102$ (second level of the Rayleigh instability spectrum in a horizontal layer, see[4]).

The behavior of the upper levels of instability is more complex. The neutral curves $R(k)$ have several extrema; their number depends on the angle and increases with the index of the level. In the limit when $\alpha = 90°$, higher levels of the Rayleigh spectrum are obtained. Figs. 6 and 7 give the neutral curves $R(k)$ for the third and fourth levels.

In conclusion we note that the change of the character of the instability during the variation of inclination is, apparently, typical for long channels of arbitrary cross section.

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